$$GH(s) = \frac{K(s+b)}{s(s+3)(s+4)} \quad \frac{K(s+a)}{s(s+b)(s+c)}$$

Root locus

(a) Poles ⟶ 3 Poles ⇒ $0, -b, -c$

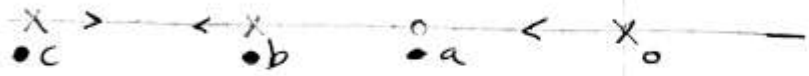
Zeros ⟶ 1 Zero ⇒ $-a$

(b) Real Part

From

$o \longrightarrow a$

$b \longrightarrow c$



مثلاً: تقف عند a وتبعد عالیمین وتشوف ال Poles (Real Part) لو فردی یبقی الجزء اللی قبلها Zero ،

لـ زی مـ a ← o

[4] Breaking Point

$1 + GH(s) = 0$ $\quad S_b = \text{--}$

$GH(s) = 0$ $\quad K \text{ at } S_b = \text{--}$

$K = \text{--}$ خطوه ال (breaking Point) إستخدمناها لحدوث تصادم بسبب الجذرين b ، c.

$\frac{dK}{ds} = 0$

5 Asymptotes

هيمشى (root locus) هى خطوط وهمية تعرفنا ال
فى أى اتجاه.

① no. of Asymptotes

$= n - m$
- n ↓ no. of poles
- m ↳ no. of zeros

② Center of Asymptotes: $C_A = \frac{\sum \text{Poles} - \sum \text{Zeros}}{n-m}$

③ $\theta = \frac{(2L+1)180}{n-m}$

Check stability

$1 + GH(s) = 0$

ex $s(s+3)(s+9) + K(s+1) = 0$

$s^3 + 7s^2 + (12+K)s + K = 0$

Routh

s^3	$1 \quad 12+K$	
s^2	$7 \quad K$	
s^1	$\frac{7(12+K)-K}{7}$	> 0 ①
s^0	K	> 0 ②

① و ② هما شروط ال (stability)

$$* \; GH(s) = \frac{(K+3)\,3}{(s+3)(s+2)}$$

نرجع لهوه $1+GH(s)$ ونفصل مابين K وال 3.

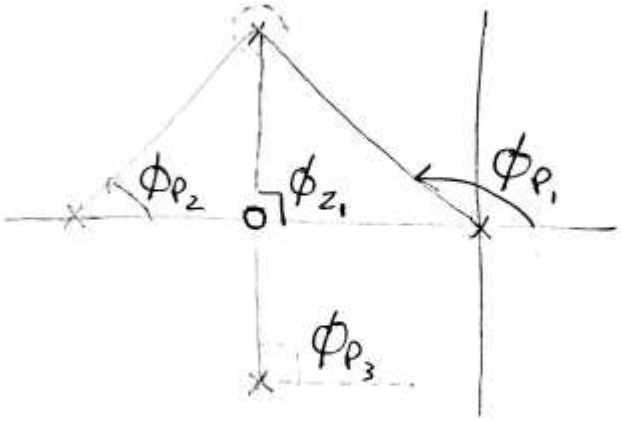
$$* \; GH(s) = \frac{20K(s+3)}{(s+4)(s+5)} \longrightarrow \text{Put } K' = 20K$$

~~Deparu~~ (Departure Angle)

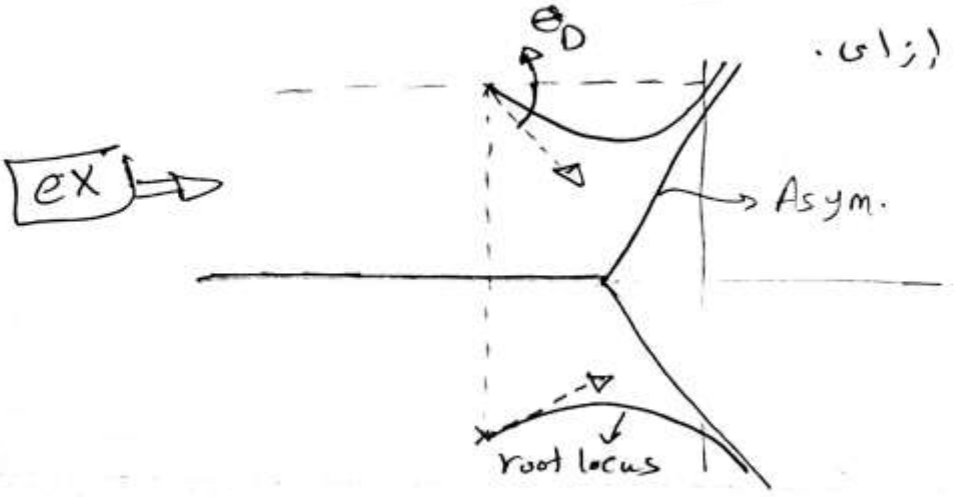
$$\Theta_D = 180 - \phi_P + \phi_Z$$

$$\phi_P = \phi_{P_1} + \phi_{P_2} + \phi_{P_3}$$

$$\phi_Z = \phi_{Z_1}$$



ـه نستخدم ال (Departure Angle) في حالة إننا عندنا (جذر تخيلي) ولا نعلم تحرك ال (root locus) منه هيكون إزاي.

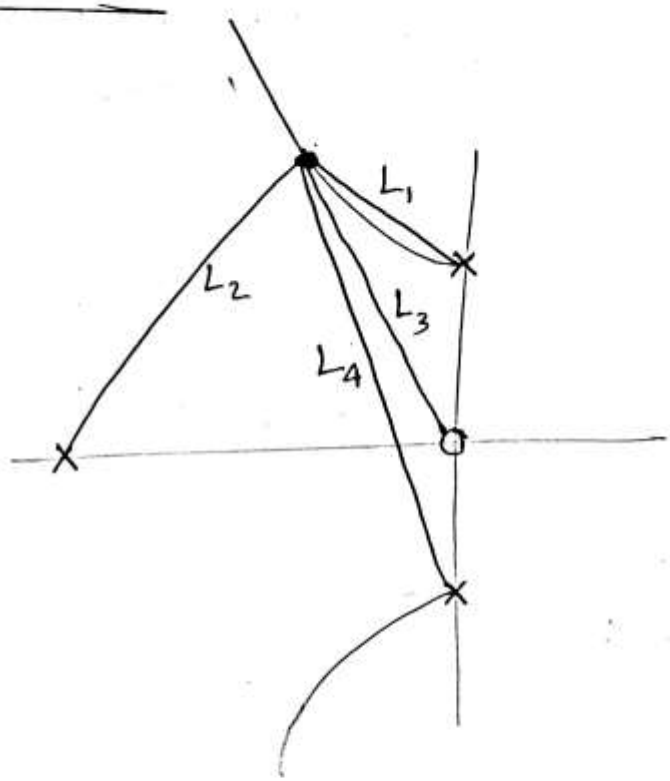


Properties

1 Find K at S_0

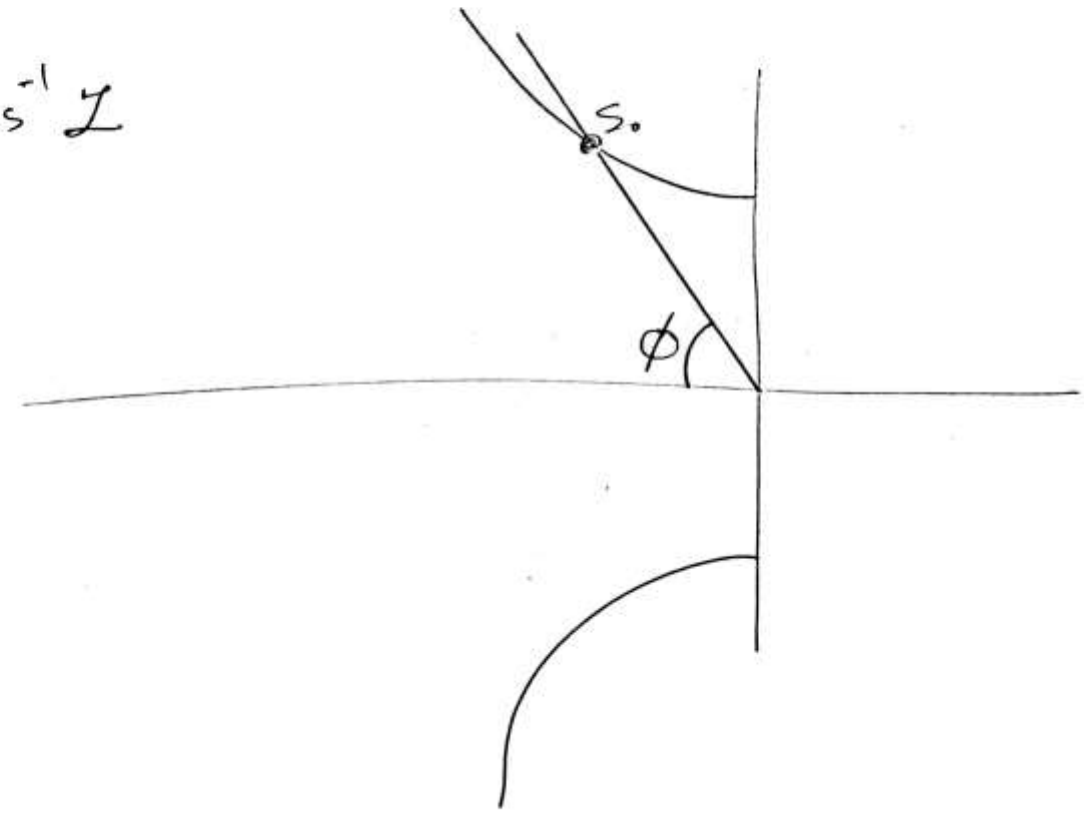
$$K\Big|_{S_0} = \frac{\Pi \text{ Poles}}{\Pi \text{ Zero}}$$

$$= \frac{L_1 L_2 L_4}{L_3}$$



2 K at $\zeta = 0.5$ (damping ratio)

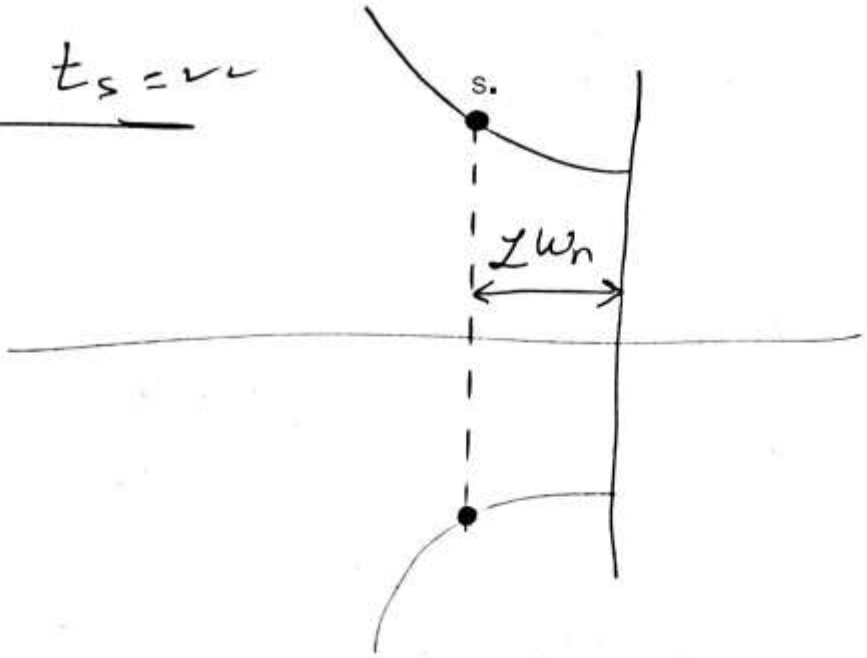
$$\phi = \cos^{-1} \zeta$$



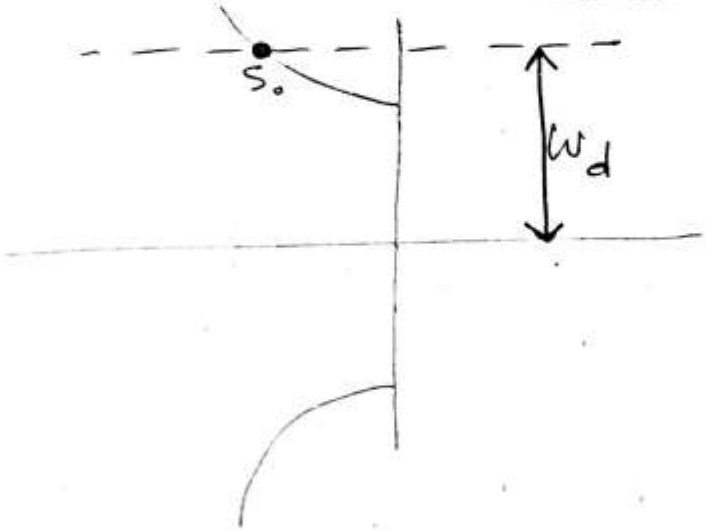
[3] Find K at t_s = ✓✓

$$t_s = \frac{4}{\zeta \omega_n}$$

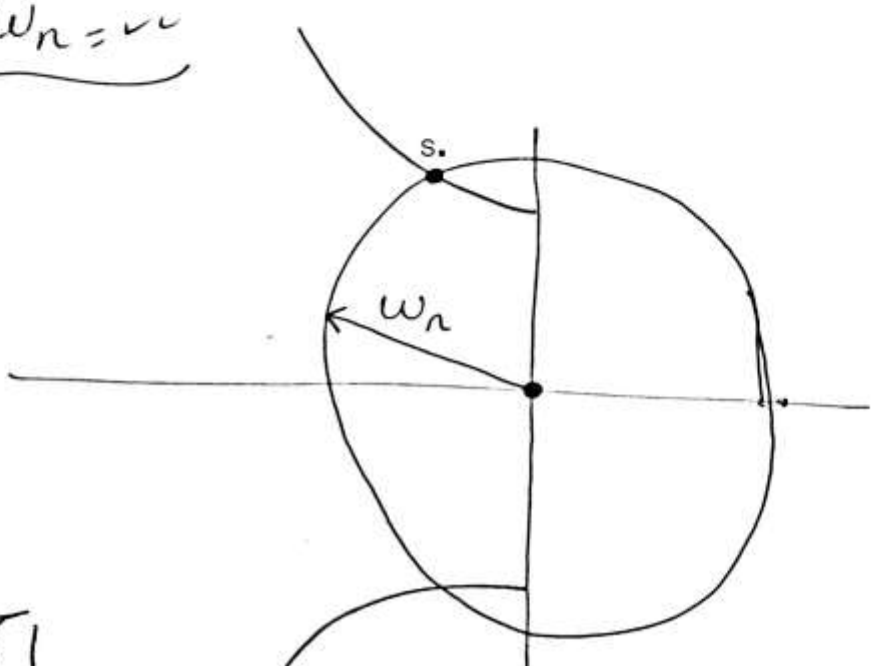
$\Rightarrow \zeta \omega_n$ = ✓✓



[4] K at ω_d = ✓✓



[5] K at ~~ω_d~~ ω_n = ✓✓



[5]

Bode diagram

given o.L.T.F $G H(s)$ to get bode diagram

① Replace $s \longrightarrow j\omega$

$$G H(j\omega) = G H(s) \Big|_{s \to j\omega}$$

② $|G H(j\omega)|$, $\angle G H(j\omega)$

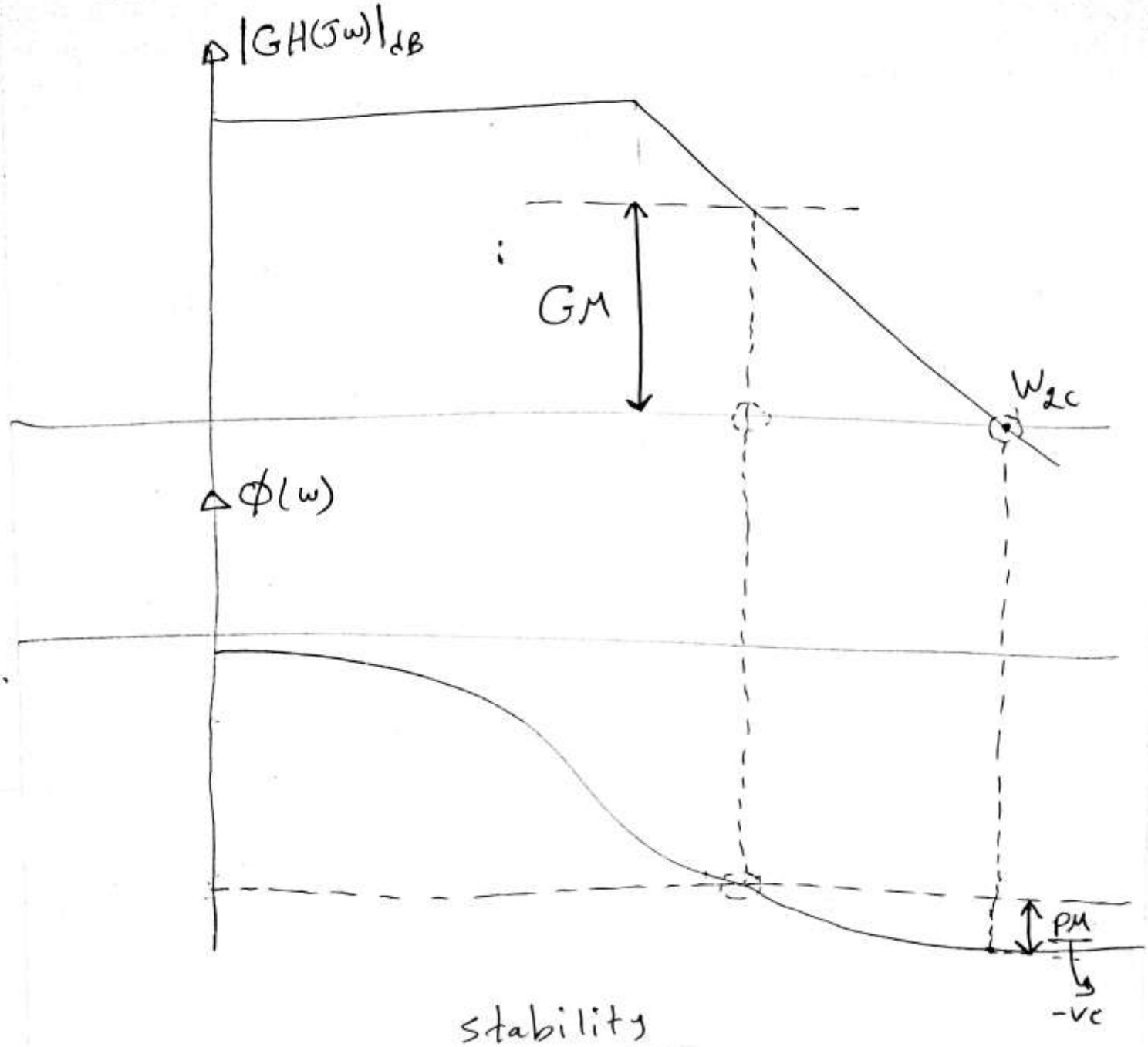
$|G H(j\omega)|$ → magnitude

$\angle G H(j\omega)$ → Angle

③ $|G H(j\omega)|_{dB} = 20 \log |G H(j\omega)|$

$$| \quad | = \sqrt{(Real)^2 + (imag)^2}$$

$$\phi = \tan^{-1}\left(\frac{imag}{Real}\right)$$



stability

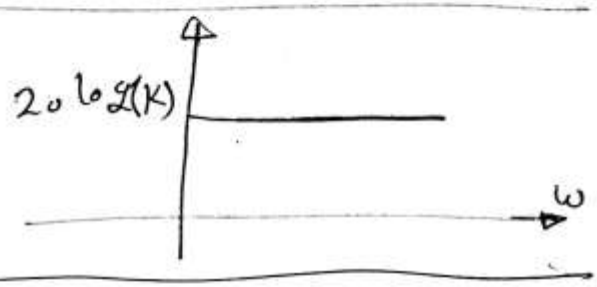
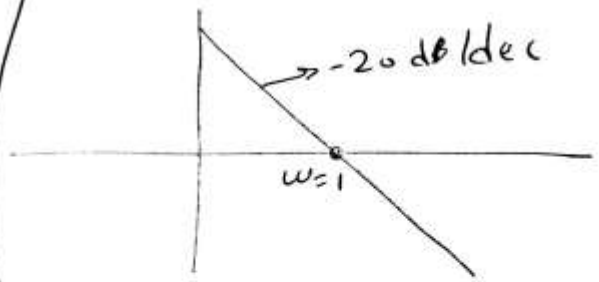
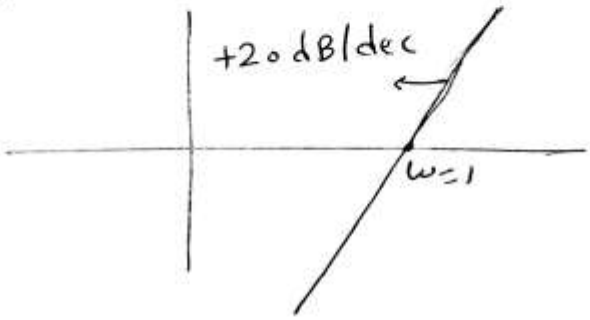
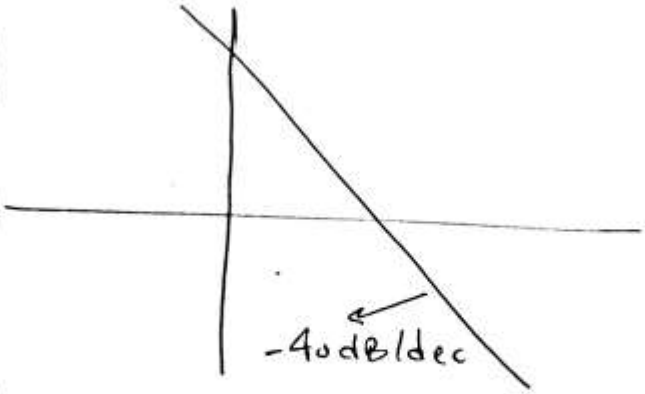
GM, PM

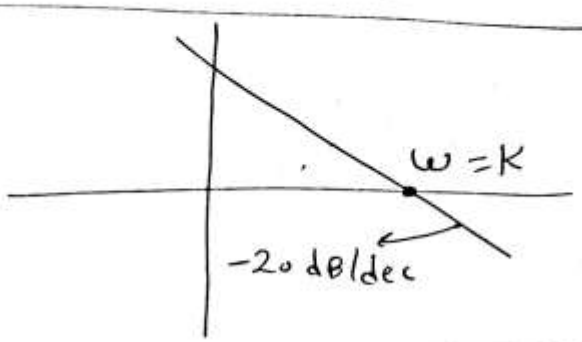
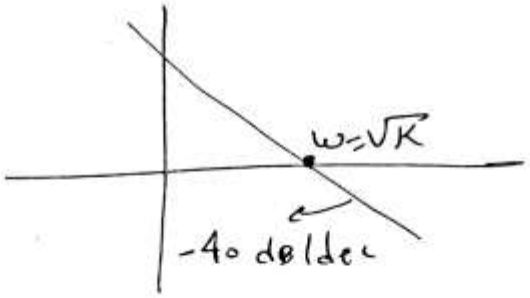
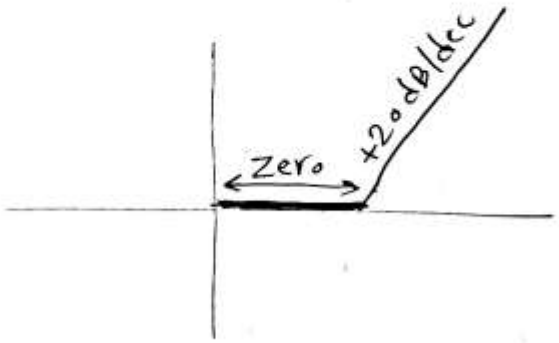
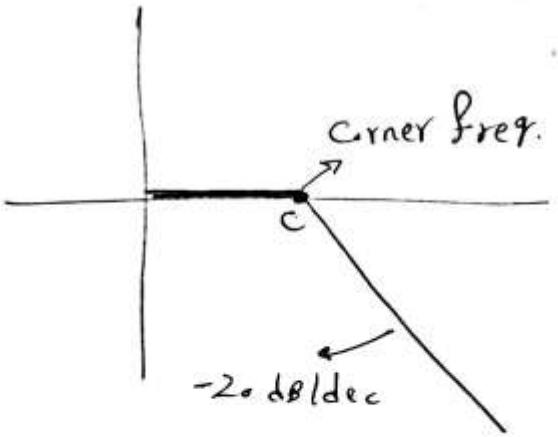
a) > 0 → +ve ⇒ stable

b) < 0 → -ve ⇒ unstable

c) $= 0$ critical stable

$$PM = 180 + \phi(\omega)\Big|_{\omega = \omega_{gc}}$$

Term	$\phi(\omega)$	$\lvert\ \rvert_{dB}$
K	0	
$\frac{1}{s}$ or $\frac{1}{j\omega}$	-90	
s or $j\omega$	$+90$	
$\frac{1}{s^2}$ or $\frac{1}{j\omega \cdot j\omega}$	-180	

Term	$\phi(\omega)$	$\lvert\lvert_{dB}$
$\frac{K}{s}$ or $\frac{K}{j\omega}$	-90	
$\frac{K}{s^2}$ or $\frac{K}{j\omega \cdot j\omega}$	-180	
$1 + \frac{s}{c}$	$\tan^{-1}\left(\frac{\omega}{c}\right)$	
$\frac{1}{1 + \frac{s}{c}}$	$-\tan^{-1}\left(\frac{\omega}{c}\right)$	

$$GH(s) = (s)^{\pm n}$$

$\hookrightarrow |\ |_{dB} \longrightarrow$ خط مستقيم يمر بـ $\boxed{\omega = 1}$

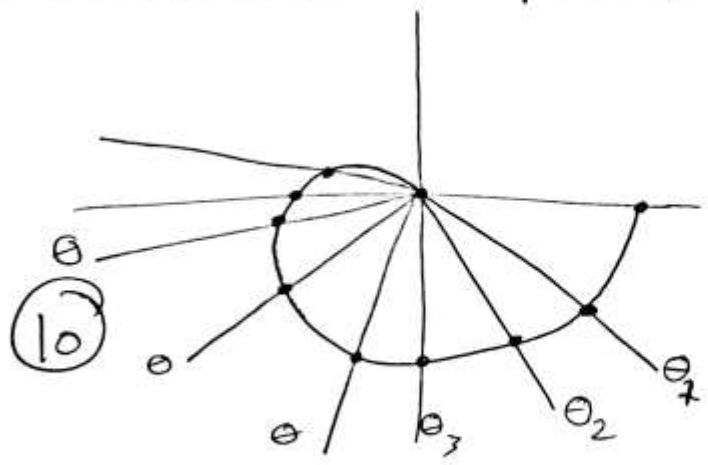
- البسط $\leftarrow \pm 20n$ dB/decade $\leftarrow$ وميله
  - $\pm$ $\leftarrow$ اشارة

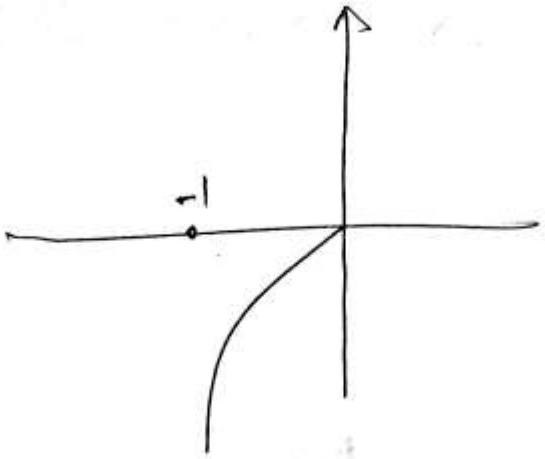
$$\phi(\omega) = \pm 90\, n$$

Polar Plot

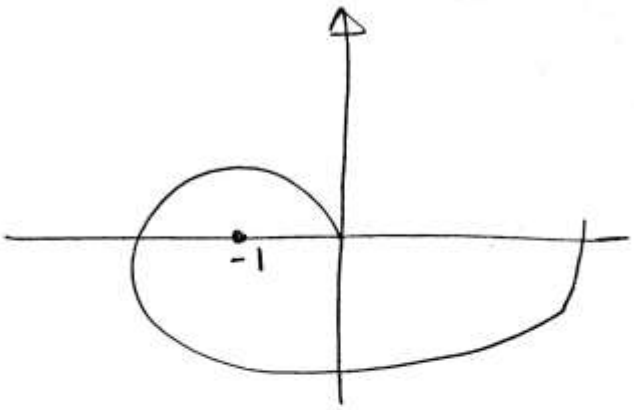
1. $s \longrightarrow j\omega \Rightarrow GH(j\omega)$
2. $|GH(j\omega)|$, $\phi(\omega) = \angle GH(j\omega)$

ω	0	. . . . .	∞
$\lvert GH(j\omega)\rvert$	:		
$\phi(\omega)$			





"stable"



"unstable"

لو اختفت بداخله (-1) يكون (unstable)

لو عايز تجيبه (GM) تنظر للجدول وتشوف أقرب زاوية لـ -180° وتقلل قيمة ω حتى تصل لـ -180°

* using try and error

$\omega = 2 \Rightarrow \phi(\omega) = -184.44$

$\omega = 1.9 \Rightarrow \phi(\omega) = -181.029$

$\omega = 1.8 \Rightarrow \phi(\omega) = -177.4^\circ$

$\omega = 1.88 \Rightarrow \phi(\omega) = -180.32^\circ$

أرقام مش شمال

$\therefore \omega_{pc} \simeq 1.88$ rad/sec

$$GM = \frac{1}{|GH(j\omega)|_{\omega=\omega_{pc}}}$$

or حل آخر (رياضي)

$$\phi(w) = -\tan^{-1}(2w) - \tan^{-1}(0.5w) - \tan^{-1}(w)$$

$w_{pc} \Longrightarrow w$ at $\phi(w) = -180$

$$-180 = -\tan^{-1}(2w) - \tan^{-1}(0.5w) - \tan^{-1}(w)$$

$$\tan(X \pm y) = \frac{\tan(X) \pm \tan(y)}{1 \mp \tan X \cdot \tan y}$$

$$180 - \tan^{-1}(2w_{pc}) = \tan^{-1}(0.5w_{pc}) - \tan^{-1}(w_{pc})$$

← نأخذ tan للطرفين

$$\frac{\tan(180) - 2w_{pc}}{1 + \tan(180)(2w_{pc})} = \frac{0.5w_{pc} + w_{pc}}{1 - 0.5w_{pc}^2}$$

$\hookrightarrow w_{pc} = 1.8708$ rad/sec

$$GM = \frac{1}{|GH(jw)|_{w=w_{pc}}}$$

PM

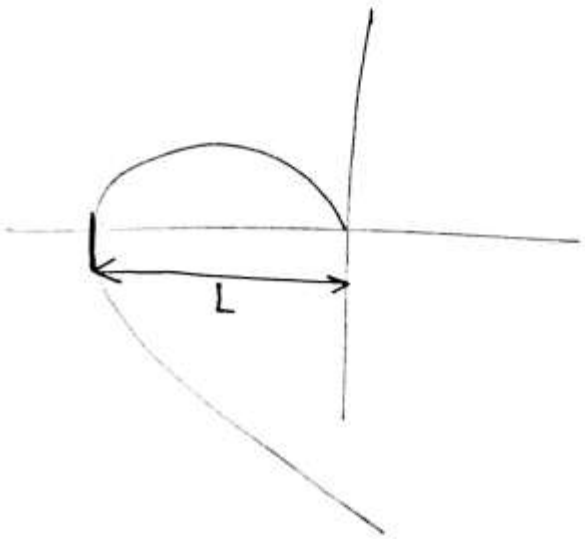
$$PM = 180 + \phi(w_{gc})$$

$|GH(jw)| = 1$ في الجدول القيمة التي تساوي

عندها قيمة w تكون w_{gc} نحسب الزاوية عندها.

$$GM = \frac{1}{L}$$

هذا أحد طرق حسابه



في الـ (PM) لو لم تجد القيمة $|GH(jw)| = 1$

وفي مثلاً القيمة 1.43 سنستخدم طريقة

((Page 11)) (try and error)

* لحد ما توصل $GH(jw)$ لقيمة قريبة من الواحد